



HAL
open science

Kinematics of nonlinear waves over variable bathymetry. Part I: Numerical modelling, verification and validation

Michel Benoit, Jie Zhang, Yuxiang Ma

► To cite this version:

Michel Benoit, Jie Zhang, Yuxiang Ma. Kinematics of nonlinear waves over variable bathymetry. Part I: Numerical modelling, verification and validation. *Coastal Engineering*, 2024, 193, pp.104577. 10.1016/j.coastaleng.2024.104577 . hal-04681767

HAL Id: hal-04681767

<https://edf.hal.science/hal-04681767v1>

Submitted on 30 Aug 2024

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

Kinematics of nonlinear waves over variable bathymetry. Part I: Numerical modelling, verification and validation

Michel Benoit^{a,b}, Jie Zhang^{c,d,*} and Yuxiang Ma^e

^aEDF R&D, Laboratoire National d'Hydraulique et Environnement (LNHE), Chatou, France

^bLHSV, Saint-Venant Hydraulics Laboratory (Ecole des Ponts, EDF R&D), Chatou, France

^cQingdao Innovation and Development Base, Harbin Engineering University, Qingdao 266400, PR China

^dQingdao Innovation and Development Center of Harbin Engineering University, Qingdao 266400, PR China

^eState Key Laboratory of Coastal and Offshore Engineering, Dalian University of Technology, Dalian 116023, PR China

ARTICLE INFO

Keywords:

Coastal waves

Nonlinear waves

Wave kinematics

Numerical simulation

Statistical parameters

ABSTRACT

Fluid particle kinematics due to wave motion (i.e. orbital velocities and accelerations) at and beneath the free surface is involved in many coastal and ocean engineering applications, e.g. estimation of wave-induced forces on structures, sediment transport, etc. This work presents the formulations of these kinematics fields within a fully nonlinear potential flow (FNPF) approach. In this model, the velocity potential is approximated with a high-order polynomial expansion over the water column using an orthogonal basis of Chebyshev polynomials of the first kind. Using the same basis, original analytical expressions of the components of velocity and acceleration are derived in this work. The estimation of particle accelerations in the course of the simulation involves the time derivatives of the decomposition coefficients, which are computed with a high-order backward finite-difference scheme in time. The capability of the numerical model in computing the particle kinematics is first validated for regular nonlinear waves propagating over a flat bottom. The model is shown to be able to predict both the velocity and acceleration of highly nonlinear and nearly breaking waves with negligible error compared to the corresponding stream function wave solution. Then, for regular waves propagating over an uneven bottom (bar-type bottom profile), the simulated results are confronted with existing experimental data, and very good agreement is achieved up to the sixth-order harmonics for free surface elevation, velocity and acceleration.

1. Introduction

Water wave kinematics has been studied for decades because it is of core importance in the design procedures of coastal and harbour engineering (e.g. Fredsøe and Deigaard, 1992). The distribution of total pressure, water particle velocities and accelerations plays a fundamental role when evaluating forces on marine structures, the motion of sediments and the evolution of coastal morphology (Freilich and Guza, 1984; Kriebel, 1998; Elfrink and Baldock, 2002; Wilson, 2002). Under the background of global warming, coastal and harbour engineering is faced with more frequent and disruptive extreme events (see e.g. Didenkulova and Pelinovsky, 2016, 2020; Didenkulova et al., 2023; Shi et al., 2024). To balance safety and economy, a better understanding and prediction of the flow field beneath strongly nonlinear waves remains of paramount importance (Zelt et al., 1995; Stansberg et al., 1995; Aggarwal et al., 2016; Vested et al., 2020; Li et al., 2023; Deng et al., 2023).

To obtain the spatial and temporal evolution of wave kinematics, deterministic (phase-resolving) models are needed. The Computational Fluid Dynamics (CFD) approach solves the Navier–Stokes equations which account for nonlinearity, vorticity and viscosity. It is very powerful in describing the flow of fluids, and in offering the pressure and velocity pro-

files. The Reynolds Averaged Navier–Stokes (RANS) equations, solved with either Lagrangian (Dalrymple and Rogers, 2006; Antuono et al., 2011) or Eulerian (Wang et al., 2009; Higuera et al., 2013; Jacobsen et al., 2015) methods, are often used in the studies of wave breaking, wave-structure interactions, and multiphase flows. One drawback of RANS equations is the high computational burden, limiting the spatial and temporal scale of the simulations. In addition, as shown recently by Larsen et al. (2019) for instance, the accuracy of computed wave kinematics is not always guaranteed with this kind of code, in particular in the vicinity of the free surface, depending on the employed numerical methods and the discretization parameters selected to solve this set of equations numerically.

Besides, there are very efficient yet simplified wave models with assumptions on the magnitudes of wave steepness and/or relative water depth, namely the weakly nonlinear and dispersive models. For instance, mild slope equation, Boussinesq equation, nonlinear Schrödinger equation, Serre-Green-Naghdi equation belong to this class (Berkhoff, 1972; Porter, 2003; Boussinesq, 1872; Madsen and Schäffer, 1998; Hasimoto and Ono, 1972; Dysthe, 1979; Green et al., 1974; Bassi et al., 2020, just to name a few in these topics). As the name indicates, models of this type are developed for handling weakly nonlinear and dispersive wave cases, i.e. in the long-wave regime. Therefore, they assume simplified representations of the profiles of horizontal and vertical velocity components in the vertical direction, typically using low-order polynomial expressions (i.e. linear or quadratic). Al-

*Corresponding author

✉ michel.benoit@edf.fr (M. Benoit); jie.zhang@hrbeu.edu.cn (J. Zhang); yuxma@dlut.edu.cn (Y. Ma)

ORCID(S): 0000-0003-4195-2983 (M. Benoit); 0000-0003-0794-2335 (J. Zhang); 0000-0003-4314-0428 (Y. Ma)

though there are numerous extensions of the above-mentioned models, all being able to include higher-order effects, they often end up in cumbersome mathematical expressions with high-order derivatives which are tricky for numerical implementation.

As a compromise between efficiency and accuracy, given the viscous and turbulent effects are often negligible for wave propagation studies, the fully nonlinear potential flow (FNPF) model represents a powerful tool for wave modelling in a range of dozens of wavelengths. In recent years, it has received considerable attention and undergone substantial progress.

On the one hand, this mathematical model makes no a priori assumption about the magnitude of wave steepness and relative water depth, thus it is capable of describing all nonlinear features of non-overtopping waves. On the other hand, it is convenient to compute the particle velocity and acceleration from the scalar velocity potential. Besides, it can be used to provide input wave fields and to couple with CFD models (Paulsen et al., 2014; Decorte et al., 2021). The FNPF model can be solved with the Boundary Element Method (BEM) that projects the problem on the boundary surface of the fluid domain using Green's identity and function (Grilli et al., 1989; Fochesato and Dias, 2006; Harris et al., 2014) or with the so-called using "Zakharov formulation" that formulates the problem with free surface variables (Zakharov, 1968; Craig and Sulem, 1993), both methods allowing for a reduction of the dimension of the computational problem. Among many approaches, the High-Order Spectral method (Dommermuth, 2000; Gouin et al., 2016; Lawrence et al., 2021a), high-order Boussinesq-type models (Madsen et al., 2006; Bingham et al., 2009) and the Hamiltonian couple-mode theory (Belibassakis and Athanassoulis, 2011; Papoutsellis et al., 2019) have been used to solve the Zakharov formulation of the FNPF model. Recently, a numerical solver of the Zakharov formulation has been developed using a highly accurate approximation of the velocity potential through the use of a spectral approach in the vertical direction with a set of orthogonal polynomials (Tian and Sato, 2008; Yates and Benoit, 2015; Raoult et al., 2016; Benoit et al., 2017). The resulting computational code, called Whispers3D (abbreviated W3D hereafter), shows excellent accuracy in predicting the evolution of highly nonlinear free surface waves over steep slopes (Zhang and Benoit, 2021; Zhang et al., 2022). The W3D code is also capable of handling wave breaking with various options for breaking initiation criteria and dissipation mechanisms (Simon et al., 2019). However, the capability of this model in describing wave kinematics at and beneath the free surface elevation (FSE) has not been discussed yet, except partially in the work of Zhang and Benoit (2021). In the present work, W3D performance in calculating the velocity and acceleration directly and explicitly is thoroughly examined, namely by first deriving analytical expressions of these kinematic fields, and then applying these new results to compute velocity and acceleration fields for highly nonlinear waves over flat and uneven bottoms.

The remainder of this article is organized as follows: in section 2, the computation formulas of particle velocity and

acceleration are derived within the model formulation; section 3 presents the verification of the method for highly nonlinear regular waves case propagating in uniform water depth against analytical results computed with the exact stream function (SF) theory; section 4 shows the experimental validation for a nonlinear regular wave shoaling and de-shoaling case over a submerged trapezoidal bar, the simulation results are compared with measurements in both spectral and statistical domain. Conclusions are provided in section 5.

2. Mathematical wave model and internal kinematics

2.1. Overview of the wave model

Within the framework of potential theory, assuming that (i) the fluid is inviscid and incompressible, (ii) the flow is irrotational, and (iii) the waves are non-breaking, the fluid motion can be described by the velocity potential ϕ . In addition, the free surface tension is neglected here, for simplicity. The above assumptions do not imply any restriction on the magnitude of the wave steepness nor on the relative water depth, thus the full nonlinearity and dispersion of waves are retained. Consider a two-dimensional case in a Cartesian coordinate system (x, z) , with z -axis pointing upward and $z = 0$ located at the still water level (SWL), the governing equations of the FNPF theory are:

$$\Delta\phi = 0 \quad \text{for } z \in [-h, \eta], \quad (1)$$

$$\eta_t + \phi_x \eta_x - \phi_z = 0 \quad \text{on } z = \eta, \quad (2)$$

$$\phi_t + \frac{1}{2} (\phi_x^2 + \phi_z^2) + g\eta = 0 \quad \text{on } z = \eta, \quad (3)$$

$$h_x \phi_x + \phi_z = 0 \quad \text{on } z = -h, \quad (4)$$

where Δ denotes the Laplace operator, $\eta(x, t)$ denotes the FSE, $h(x)$ the water depth, and g the acceleration due to gravity. Partial derivatives are denoted with subscripts (e.g. $\eta_x \equiv \partial\eta/\partial x$).

Following Zakharov (1968) and Craig and Sulem (1993), the FNPF problem can be reformulated as a functional of two free surface variables, i.e., the FSE $\eta(x, t)$ and the free surface potential $\tilde{\phi}(x, t) \equiv \phi(x, z = \eta, t)$. The reformulation allows describing the motion of the bulk fluid by rewriting the nonlinear free surface boundary conditions (2) and (3) as:

$$\eta_t = -\tilde{\phi}_x \eta_x + \tilde{w} \left[1 + (\eta_x)^2 \right], \quad (5)$$

$$\tilde{\phi}_t = -g\eta - \frac{1}{2} (\tilde{\phi}_x)^2 + \frac{1}{2} \tilde{w}^2 \left[1 + (\eta_x)^2 \right], \quad (6)$$

where $\tilde{w}(x, t) \equiv \phi_z(x, z = \eta, t)$ denotes the vertical velocity of the water particles on the free surface. Partial derivatives of $\tilde{\phi}$ are deduced via the chain rule. The two coupled equations (5–6) describe the change rate of $\tilde{\phi}$ and η in time, with \tilde{w} being the only unknown variable. Solving \tilde{w} from the boundary conditions $(\eta, \tilde{\phi})$ on the free surface is the so-called *Dirichlet-to-Neumann* (DtN) problem. The DtN problem is of fundamental importance for the Zakharov

166 formulation and has been extensively studied. For a more detailed
167 introduction of various approaches for solving the DtN
168 problem in different models and the methodology adopted in
169 W3D, the readers are referred to Tian and Sato (2008); Yates
170 and Benoit (2015); Raoult et al. (2016), and the references
171 therein. Here, we first briefly review the key steps of solving
172 the DtN problem in W3D, then introduce the calculation of
173 internal kinematics in this model.

First, the physical (x, z) domain with variable boundaries in the vertical direction $z \in [-h(x), \eta(x, t)]$ is mapped to a new (x, s) domain with fixed boundaries $s \in [-1, 1]$ via the transformation of vertical coordinate:

$$s(x, z, t) = \frac{2z + h^-(x, t)}{h^+(x, t)} \quad (7)$$

174 with $h^\pm(x, t) \equiv h(x) \pm \eta(x, t)$. The potential is expressed as
175 $\varphi(x, s(x, z, t), t) \equiv \phi(x, z, t)$ in the transformed domain, in
176 which the governing equations are reformulated (not shown
177 here, see e.g. (Yates and Benoit, 2015)).

Then, the velocity potential is projected onto an orthogonal polynomial basis formed by the set of Chebyshev polynomials of the first kind, denoted $T_n(s)$, up to a maximal order denoted N_T . The potential φ is approximated as:

$$\varphi(x, s, t) \approx \varphi_{N_T}(x, s, t) = \sum_{n=0}^{N_T} a_n(x, t) T_n(s), \quad (8)$$

178 where the coefficients $a_n(x, t)$, $n = 0, 1, \dots, N_T$, are the $N_T + 1$
179 unknowns of the problem (at each abscissa x). The N_T
180 parameter plays a central role in balancing model accuracy
181 and computational effort. Benoit et al. (2017) showed the
182 capability of the linearized W3D model in describing the linear
183 dispersion, which serves as a good starting point in calibrating
184 the value of N_T . Usually, for waves with intermediate
185 nonlinearity propagating in finite water depth, choosing
186 $N_T = 7$ yields already very accurate predictions. We may
187 need to tune N_T for higher values when considering highly
188 nonlinear waves (close to breaking) or very deep water
189 conditions ($\mu \sim 10$). Inserting eq. (8) into the Laplace equation
190 and adopting the Chebyshev-tau method to remove the dependency
191 on the s variable, $N_T - 1$ linear equations are built.
192 With two supplementary equations drawn from the free surface
193 and bottom boundary conditions, the problem is closed
194 with $N_T + 1$ linear equations for $N_T + 1$ unknowns.

Then, \tilde{w} can be evaluated as:

$$\tilde{w}(x, t) = \frac{2}{h^+(x, t)} \sum_{n=1}^{N_T} a_n(x, t) n^2. \quad (9)$$

195 The DtN problem is thus solved and, with \tilde{w} known, eqs. (5–
196 6) can be stepped forward in time. Several time marching
197 schemes are implemented in W3D. Here we used the explicit
198 third-order strong stability preserving Runge-Kutta scheme
199 with a constant time-step Δt . Unless otherwise stated, all
200 first- and second-order x -derivatives in the numerical model
201 are approximated with finite difference (FD) formulas using
202 a centered stencil of $N_{sten} = 5$ nodes, which provides fourth-
203 order formal accuracy in the case of uniform grid size Δx .

2.2. Computation of orbital velocities

204 With the a_n coefficients determined, the orbital velocities
205 can be evaluated at any point in the fluid domain by taking
206 the spatial derivatives of $\phi(x, z, t)$. To that end, we first
207 give the expressions of the partial derivatives of the vertical
208 coordinate $s(x, z, t)$, obtained from eq. (7), as:
209

$$s_x = \frac{h_x^- - s h_x^+}{h^+} = \frac{h_x^-}{h^+} (1 - s) - \frac{\eta_x}{h^+} (1 + s), \quad (10)$$

$$s_z = \frac{2}{h^+}, \quad (11)$$

$$s_t = \frac{h_t^- - s h_t^+}{h^+} = -\frac{\eta_t}{h^+} (1 + s), \quad (12)$$

$$s_{xt} = -\frac{1}{h^+} [\eta_{xt} (1 + s) + \eta_t s_x + s_t h_x^+], \quad (13)$$

$$s_{zt} = -\frac{2\eta_t}{(h^+)^2}. \quad (14)$$

The horizontal velocity $u(x, z, t) = \phi_x$ and the vertical
210 velocity $w(x, z, t) = \phi_z$ are expressed as functions of the a_n
211 coefficients and their x -derivatives:
212

$$u(x, z, t) \approx \frac{\partial \varphi_{N_T}}{\partial x} + s_x \frac{\partial \varphi_{N_T}}{\partial s} \quad (15)$$

$$= \sum_{n=0}^{N_T} a_{n,x} T_n + \frac{h_x^- - s h_x^+}{h^+} \sum_{n=1}^{N_T} a_n T_{n,s},$$

$$w(x, z, t) \approx s_z \frac{\partial \varphi_{N_T}}{\partial s} = \frac{2}{h^+} \sum_{n=1}^{N_T} a_n T_{n,s}. \quad (16)$$

213 These expressions can be projected on the basis formed
214 by the orthogonal polynomials $\{T_n, n = 0, \dots, N_T\}$ using the
215 inner product defined in Eq. (24) of Raoult et al. (2019), and
216 equivalently reformulated in a compact form, as:

$$u(x, z, t) = \sum_{p=0}^{N_T} a_p^u T_p, \quad (17)$$

$$w(x, z, t) = \sum_{p=0}^{N_T-1} a_p^w T_p, \quad (18)$$

with the coefficients a_p^u , $p = 0, \dots, N_T$, and a_p^w , $p = 0, \dots, N_T - 1$,
217 given analytically by:
218

$$a_p^u = a_{p,x} + \frac{1}{h^+} (h_x^- S_{1p} - h_x^+ S_{2p}), \quad (19)$$

$$a_p^w = \frac{2}{h^+} S_{1p}. \quad (20)$$

where

$$S_{1p} \equiv \sum_{n=1}^{N_T} a_n B_{p01n}, \quad (21)$$

$$S_{2p} \equiv \sum_{n=1}^{N_T} a_n B_{p11n}. \quad (22)$$

220 The expressions of the terms B_{p01n} and B_{p11n} are already
 221 defined in appendix B of Raoult et al. (2019), thus not re-
 222 peated here for brevity. We note the maximum degree of T_n
 223 polynomials is N_T for u in eq. (17) and $N_T - 1$ for w in
 224 eq. (18).

2.3. Computation of Eulerian accelerations

225 The local (Eulerian) accelerations $ax(x, z, t) = u_t$ and
 226 $az(x, z, t) = w_t$ are obtained as:

$$ax \approx \sum_{n=0}^{N_T} a_{n,xt} T_n + \sum_{n=1}^{N_T} [s_t a_{n,x} + s_{xt} a_n + s_x a_{n,t}] T_{n,s} \quad (23)$$

$$+ s_x s_t \sum_{n=2}^{N_T} a_n T_{n,ss},$$

$$az \approx \sum_{n=1}^{N_T} (s_z a_{n,t} + s_{zt} a_n) T_{n,s} + s_z s_t \sum_{n=2}^{N_T} a_n T_{n,ss} \quad (24)$$

228 As for the velocity components, these expressions can be
 229 projected on the basis $\{T_n, n = 0, \dots, N_T\}$ and equivalently
 230 reformulated as:

$$ax(x, z, t) = \sum_{p=0}^{N_T} a_p^{ax} T_p, \quad (25)$$

$$az(x, z, t) = \sum_{p=0}^{N_T-1} a_p^{az} T_p, \quad (26)$$

231 with the coefficients $a_p^{ax}, p = 0, \dots, N_T$, and $a_p^{az}, p = 0, \dots, N_T -$
 232 1, given analytically by:

$$a_p^{ax} = a_{p,xt} - \frac{2}{h^+} \left[\eta_t (S_{3p} + S_{4p}) + (\eta_{xt} - 2 \frac{\eta_t \eta_x}{h^+}) \times \right. \quad (27)$$

$$\left. (S_{1p} + S_{2p}) - 2 \frac{\eta_t h_x}{h^+} S_{2p} - h_x^- S_{5p} + h_x^+ S_{6p} \right]$$

$$+ \frac{\eta_t}{(h^+)^2} [h_x (S_{9p} - S_{7p}) + \eta_x (S_{7p} + 2S_{8p} + S_{9p})],$$

$$a_p^{az} = \frac{2}{h^+} (S_{5p} - \frac{\eta_t}{h^+} (S_{1p} + S_{7p} + S_{8p})), \quad (28)$$

233 where

$$S_{3p} \equiv \sum_{n=1}^{N_T} a_{n,x} B_{p01n} = \frac{\partial S_{1p}}{\partial x}, \quad (29)$$

$$S_{4p} \equiv \sum_{n=1}^{N_T} a_{n,x} B_{p11n} = \frac{\partial S_{2p}}{\partial x}, \quad (30)$$

$$S_{5p} \equiv \sum_{n=1}^{N_T} a_{n,t} B_{p01n} = \frac{\partial S_{1p}}{\partial t}, \quad (31)$$

$$S_{6p} \equiv \sum_{n=1}^{N_T} a_{n,t} B_{p11n} = \frac{\partial S_{2p}}{\partial t}, \quad (32)$$

$$S_{7p} \equiv \sum_{n=1}^{N_T} a_n B_{p02n}, \quad (33)$$

$$S_{8p} \equiv \sum_{n=1}^{N_T} a_n B_{p12n}, \quad (34)$$

$$S_{9p} \equiv \sum_{n=1}^{N_T} a_{n,t} B_{p22n}. \quad (35)$$

234 The expressions of the terms B_{p02n} , B_{p12n} and B_{p22n} are also
 235 provided in Raoult et al. (2019).

236 To compute the accelerations, we need to provide the
 237 time derivatives of the FSE, denoted η_t (appearing in eqs. (12)–
 238 (14)), and those of the a_n coefficients, denoted $a_{n,t}$ (appear-
 239 ing in eqs. (23)–(24)). For η_t , we use the expression at the
 240 right-hand side of eq. (5). Then, η_{xt} can be obtained from η_t
 241 with the 5-node centered FD scheme mentioned above.

242 The $a_{n,t}$ terms are computed using a high-order FD scheme
 243 in time, with two options available:

- calculation of kinematics after completion of the sim- 244
 ulation: a centered scheme around the current time 245
 instant is used, with again a 5-node stencil: 246

$$a_{n,t}(t) = \frac{1}{12\Delta t} [8 (a_n(t + \Delta t) - a_n(t - \Delta t)) - (a_n(t + 2\Delta t) - a_n(t - 2\Delta t))] \quad (36)$$

- calculation in the course of the simulation: an upwind 247
 scheme is used, with the current value of a_n and the 248
 three previous ones (giving third-order formal accu- 249
 racy): 250

$$a_{n,t}(t) = \frac{1}{\Delta t} \left[\frac{11}{6} a_n(t) - 3a_n(t - \Delta t) + \frac{3}{2} a_n(t - 2\Delta t) - \frac{1}{3} a_n(t - 3\Delta t) \right]. \quad (37)$$

251 As for η_{xt} , the terms $a_{n,xt}$ are then computed from the terms
 252 $a_{n,t}$ using again a centered 5-node FD scheme in space.

3. Verification for regular nonlinear waves in uniform water depth

253 In this section, we demonstrate the accuracy of the pro-
 254 posed formulas for computing wave kinematics of a highly
 255 nonlinear periodic wave propagating with permanent form in
 256 uniform water depth. For a quantitative assessment, the nu-
 257 merical solution of SF theory is considered, which provides
 258 a solution of arbitrarily high accuracy for this case. The SF
 259 solution is obtained here by imposing that the mean Eulerian
 260 flow velocity at any point below the wave trough is null.
 261

262 Given the still water depth h , wavelength L (or, equiva-
 263 lently, wave number $k = 2\pi/L$), and wave height H , a regu-
 264 lar wave condition is defined. Here, we choose a challeng-
 265 ing case: (i) the relative water depth is set to $h/L = 1$ (i.e.
 266 $kh = 2\pi$), achieving twice the traditionally accepted ‘deep
 267 water’ threshold ($kh = \pi$); (ii) a very high value of the wave
 268

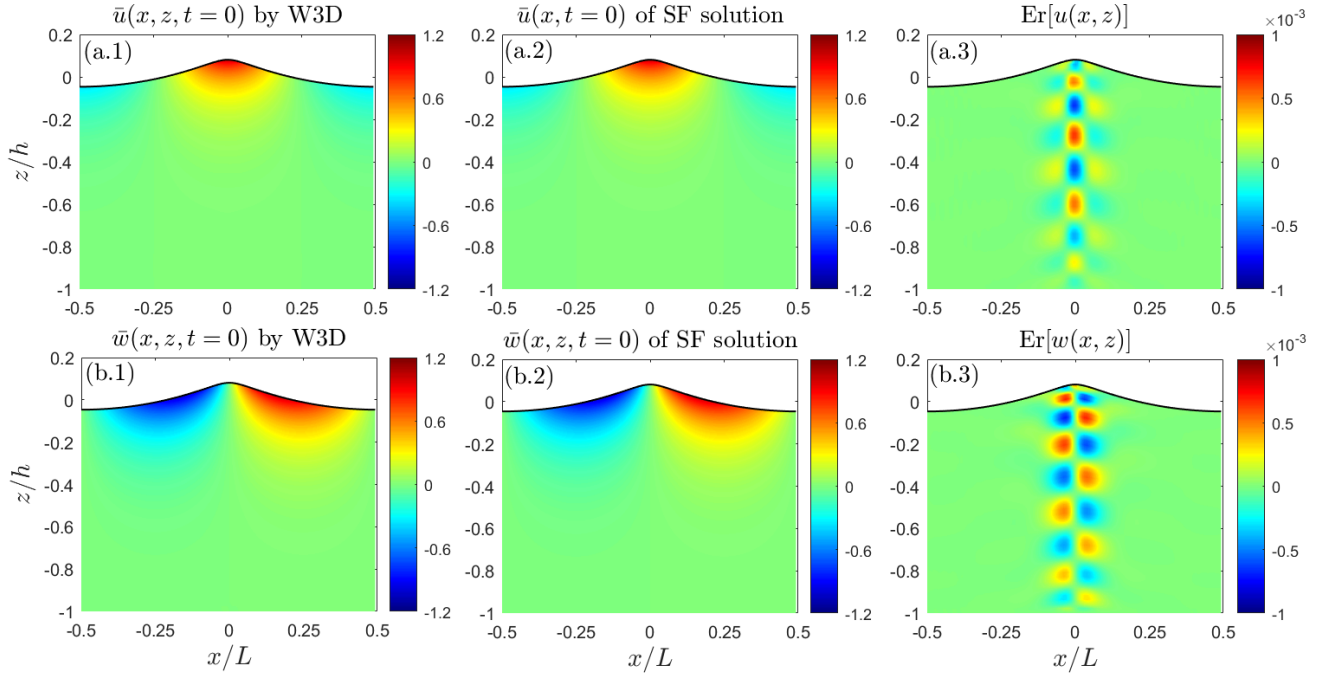


Figure 1: Normalized velocity fields (\bar{u}, \bar{w}) at $t = 0$ predicted by the W3D model in panels (a.1) and (b.1) and the corresponding SF solution in panels (a.2) and (b.2), respectively, and the relative error of the velocities obtained with two methods in panels (a.3) and (b.3). In each panel, the wave profile is outlined with a thick black line on the free surface.

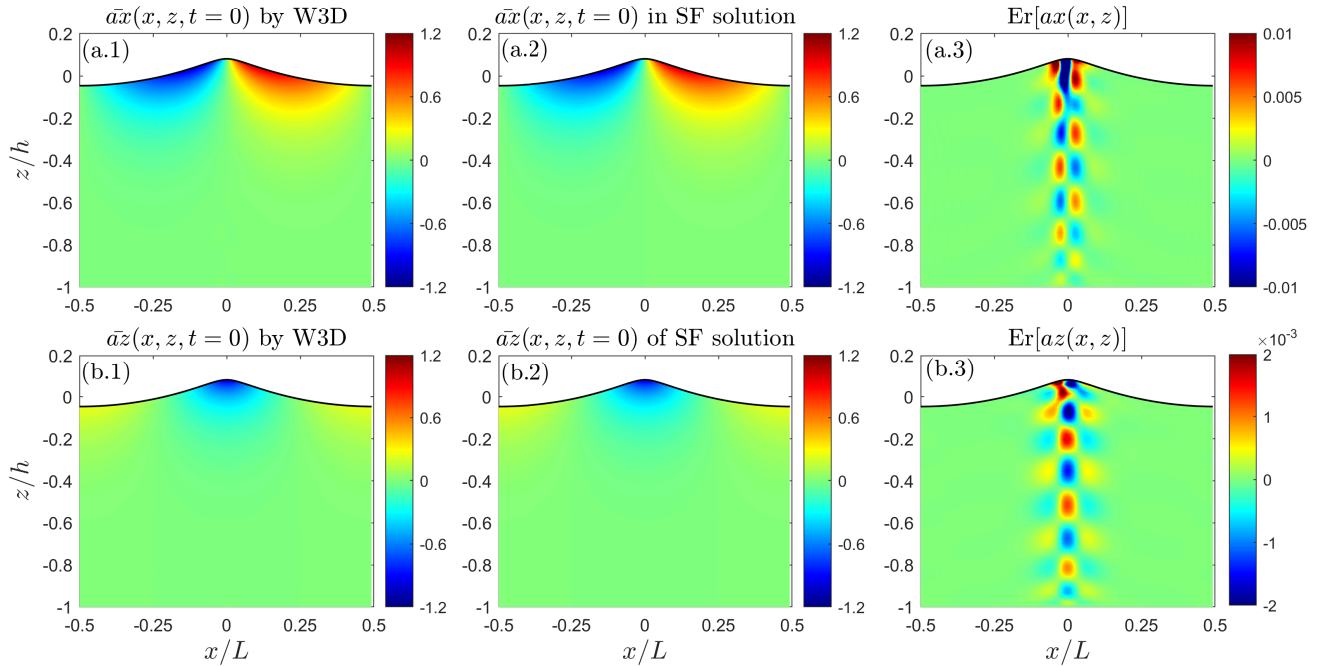


Figure 2: Normalized acceleration fields (\bar{a}_x, \bar{a}_z) at $t = 0$ predicted by the W3D model in panels (a.1) and (b.1) and the corresponding SF solution in panels (a.2) and (b.2), respectively, and the relative error of the accelerations obtained with two methods in panels (a.3) and (b.3). In each panel, the wave profile is outlined with a thick black line on the free surface.

269 steepness is selected $H/L = 12.73\%$ (i.e. $kH/2 = 0.40$).
 270 According to the approximate relationship giving the maximum
 271 stable wave height, expressed as eq. (32) of Fenton
 272 (1990), such a wave steepness represents 90% of the maximum
 273 wave height for the chosen relative water depth. In

practice, the wavelength is set as $L = 64$ m. We use 25
 Fourier coefficients for the expansion of the SF, providing a
 converged solution. 274 275 276

The spatial profiles of FSE (η) and free surface potential
 ($\tilde{\phi}$) computed with the SF method are given as initial

conditions to the W3D simulation. We run the W3D simulation over a periodic domain covering one wavelength exactly, which is discretized with 128 cells of constant size $\Delta x = L/128 = 0.5$ m. The theoretical wave period obtained from the SF solution is $T_{\text{SF}} \equiv 2\pi/\omega_{\text{SF}} \approx 5.916$ s. We note that, as expected, this period is shorter than the period obtained from the dispersion relation of linear waves:

$$T_{\text{lin}} = \frac{2\pi}{\omega_{\text{lin}}} = \frac{2\pi}{\sqrt{gk \tanh(kh)}} \approx 6.402 \text{ s.} \quad (38)$$

We note T_{SF} is smaller than T_{lin} because the SF wave solution follows a nonlinear dispersion relation which predicts a faster wave phase speed for a wave with the same wavelength in comparison to the prediction of linear dispersion. In the simulation, a constant time step $\Delta t = T_{\text{SF}}/128 \approx 0.046$ s is chosen, which results in a Courant–Friedrichs–Lewy number $\text{CFL} \equiv (L\Delta t)/(T_{\text{SF}}\Delta x) = 1$. Initial conditions for $(\eta(x), \tilde{\phi}(x))$ at $t = -3\Delta t$ from the SF solution are provided as input to W3D, and time integration is performed over $3\Delta t$ to reach $t = 0$ s where results are compared with the SF solution. As kh is quite large here and waves are strongly nonlinear, we choose $N_T = 11$ in the W3D simulation after calibration.

The computed velocities and accelerations are presented hereafter in a dimensionless way, normalized with the modulus of the corresponding quantities expressed from linear wave theory (under the same condition (h, L, H) as for the SF solution) at mean water level (i.e., $z = 0$), that is $\bar{u} \equiv u/U_{\text{lin}}$, $\bar{w} \equiv w/W_{\text{lin}}$, $\bar{ax} \equiv ax/Ax_{\text{lin}}$, $\bar{az} \equiv az/Az_{\text{lin}}$, with:

$$U_{\text{lin}} = gka_{\text{lin}}/\omega_{\text{lin}} \quad (39)$$

$$W_{\text{lin}} = a_{\text{lin}}\omega_{\text{lin}} \quad (40)$$

$$Ax_{\text{lin}} = gka_{\text{lin}} \quad (41)$$

$$Az_{\text{lin}} = a_{\text{lin}}\omega_{\text{lin}}^2 \quad (42)$$

with $a_{\text{lin}} \equiv H/2$. The relative errors are made dimensionless by the maximum value of the modulus of the SF solution, e.g. for the vertical velocity component:

$$\text{Er}[w(x, z)] = \frac{w_{\text{W3D}}(x, z) - w_{\text{SF}}(x, z)}{|w_{\text{SF}}(x, z = \eta)|_{\text{max}}}. \quad (43)$$

The non-dimensional velocity fields (\bar{u} and \bar{w}) at $t = 0$ (with wave crest at $x = 0$) in both W3D simulation and SF theory are displayed in Fig. 1, with their relative errors provided. As shown in Fig. 1(a.1–a.2) and (b.1–b.2), the velocity fields (\bar{u} , \bar{w}) are well predicted by the W3D model, with the coloured maps of theoretical and simulated velocity components being visually indistinguishable from each other. The plots (a.3–b.3) of relative error show the differences appear mainly below the wave crest and remain less than 0.1% all over the water column. The velocity field underneath the wave trough is well predicted, with the maximum relative error below 10^{-4} .

The acceleration at $t = 0$ is also computed, which involves the backward FD scheme (37) to evaluate the time derivative of the coefficients $a_n(x, t = 0)$. Fig. 2 shows the

non-dimensional acceleration fields (\bar{ax} and \bar{az}) at $t = 0$ in both W3D simulation and SF theory, as well as the relative error between them. Again, the simulated and theoretical acceleration (x, z) -maps are visually identical in the current colour scale of panels (a.1–a.2) and (b.1–b.2) in Fig. 2. The relative error of ax in panel (a.3) achieves the most prominent values beneath the wave crest. It is larger than the ones reached for the velocity components but remains bounded by 1%. The relative error is lower for the vertical acceleration az in panel (b.3), not exceeding 0.5%. These low error levels confirm the accuracy of the scheme chosen to evaluate $a_{n,t}$, bearing in mind that the model has evolved the input initial solution for 3 time-steps to reach the state shown in Fig. 2.

In conclusion, the W3D schemes can model with accuracy the wave kinematics beneath highly nonlinear (close-to-breaking) regular waves, without any sign of singularity. Larger errors are observed below the wave crest (never exceeding 0.1% for the velocity components and 1% for the acceleration components in the case shown here). It is also observed that the relative errors of the horizontal components of velocity and acceleration are a bit higher than those of their vertical counterparts.

4. Experimental validation for regular nonlinear waves in variable water depth

4.1. Experimental configuration

In this section, W3D's performance is evaluated by simulating an experimental test for which detailed measurements of the FSE and orbital velocity beneath regular waves are available. We chose a case with regular nonlinear waves propagating over an inhomogeneous medium performed at the hydrodynamics laboratory of the Department of Mathematics of the University of Oslo (Norway) and introduced in Lawrence et al. (2021a). The wave flume is 24.6 m long and 0.5 m wide. A trapezoidal-shaped bar was installed on the bottom, consisting of plane ascending and descending slopes with 1/3.81 gradient and 1.6 m length each, and a plateau of 1.6 m length that connects the two slopes. The water depth is changed from $h_1 = 0.53$ m before and after the bar to $h_2 = 0.11$ m atop the bar, so that the height of the bar above the horizontal seabed is 0.42 m (see Fig. 3). A piston-type wavemaker is located at one end of the flume, and a wave absorbing zone at the other. Given the origin of the x -axis set at the beginning of the bar crest, the abscissa of the wavemaker is -12.36 m.

The measuring devices contain four ultrasonic wave probes to record the FSE, and one Nortek "Vectrino" acoustic Doppler velocimeter (ADV) to record the velocity at an elevation $z_0 = -0.05$ m below the SWL, giving $z_0 \approx -0.1h_1 \approx -0.45h_2$. The test with regular waves presented in Section 3.4 of Lawrence et al. (2021a) is considered here. In this test, the wave frequency is set as $f_0 = 0.7$ Hz (i.e., $T_0 \approx 1.43$ s, the corresponding wavelength before the bar is $L_1 = 2\pi/k_1 = 2.69$ m according to the linear wave dispersion relation) and the wave amplitude $H_0 = 2a_0 = 0.0270$ m. In such a configuration, the incident wave steepness is $\epsilon_1 \equiv k_1a_0 =$

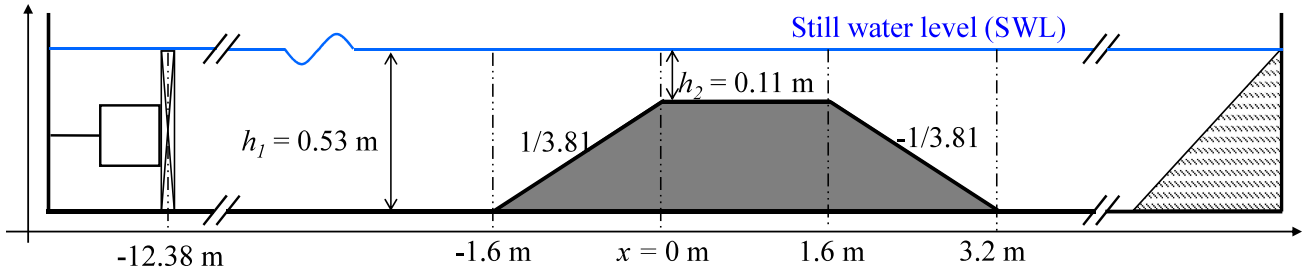


Figure 3: Sketch of the bathymetry used in the experiments reported in Lawrence et al. (2021a)

366 0.032 (or $H_0/h_1 = 0.051$) and the relative water depth $\mu_1 \equiv$
 367 $k_1 h_1 = 1.237$ (or $h_1/L_1 = 0.197$), resulting in an Ursell
 368 number $Ur_1 \equiv (H_0/h_1) (L_1/h_1)^2 = 13.12$. Over the bar,
 369 the parameters are then $\epsilon_2 = 0.059$ (or $H_0/h_2 = 0.246$),
 370 $\mu_2 = 0.483$ (or $h_2/L_2 = 0.077$) and $Ur_2 = 414.93 >$
 371 100 . Therefore the relative importance of nonlinearity is
 372 considerably enhanced by the shoal. The wave field near the
 373 wavemaker is of intermediate nonlinearity, yet it results in
 374 highly nonlinear but non-breaking waves over the bar. The
 375 same configuration was tested 31 times, such that the FSE
 376 was measured at 124 positions and the horizontal velocity at
 377 31 positions. The measurements were performed in a suffi-
 378 ciently short time to avoid the effects of reflection at the end
 379 of the flume.

380 4.2. Numerical model setup

381 In the numerical flume, the waves are generated and damped
 382 within two relaxation zones of 8 m in length (approximately
 383 $3L_1$) located at both ends of the numerical flume. The effec-
 384 tive computation domain for wave propagation (i.e., exclud-
 385 ing the two relaxation areas) starts at $x = -5.5$ m and ends
 386 at $x = 10.3$ m. The governing equations are discretized with
 387 constant space and time step, $\Delta x = 0.04$ m and $\Delta t = 0.02$ s,
 388 respectively. Such a choice results in CFL= 0.94 in the
 389 deeper flat regions and CFL= 0.50 over the bar crest. The
 390 polynomial order is set to $N_T = 7$.

391 4.3. Free surface elevation

392 Comparisons of the measured and computed FSE time
 393 profiles are shown in Fig. 4 at 6 positions over a time win-
 394 drow of $3T_0$. Before the bar (probes #1 and #25), the waves
 395 are nearly symmetric in both the horizontal and vertical di-
 396 rections. As waves propagate over the bar (probes #70 and
 397 #106), the wave profiles become asymmetric, and secondary
 398 crests manifest. These secondary crests are related to the
 399 development of high-order harmonics that propagate with
 400 a different velocity in comparison to the carrier wave. As
 401 waves propagate over the de-shoaling slope (probes #115
 402 and #124), due to the presence of bound and free super-
 403 harmonics (propagating with different velocities) the free sur-
 404 face time profile is very variable from one position to an-
 405 other. It should be noticed that after the de-shoaling slope
 406 (probe #124), the wave profile is asymmetric, with sharper
 407 troughs and flatter crests. The evolution of the wave pro-
 408 file along the flume is very well reproduced by the model,

409 despite some slight phase shifts at probes #115 and #124.

410 To better illustrate the evolution of the waves over the
 411 varying bathymetry and to assess the magnitude of nonlinear
 412 effects, a Fourier analysis is applied to both the measured and
 413 simulated time series. Fig. 5(a) shows the spatial evolution
 414 of the amplitudes of the first six harmonics (i.e. wave com-
 415 ponents with frequencies nf_0 , with $n = 1, \dots, 6$), normal-
 416 ized by the amplitude measured at probe #1, denoted as a_0 . The
 417 fourth to sixth harmonics are duplicated in Fig. 5(b) with a
 418 reduced range in y-axis to have a better view. The evolution
 419 of the primary component shows some oscillations before
 420 and over the bar, this is expected to be the result of reflection
 421 (by both the ascending and descending slopes). Over the bar
 422 crest, the amplitude of the primary component starts to de-
 423 crease due to the enhancement of the amplitudes of the high-
 424 order super-harmonics. The second harmonic is consider-
 425 ably increased over the bar crest and remains on a high level.
 426 Eventually, the amplitude of this second harmonic becomes
 427 comparable to the primary component over the de-shoaling
 428 slope. The third harmonic is also increased over the bar
 429 with some oscillations, and it decreases over the de-shoaling
 430 slope. The evolution trends of the first three harmonics are
 431 very closely reproduced by the model. In Fig. 5(b), the evo-
 432 lution trend of the fourth to sixth harmonics is similar to that
 433 of the third harmonics and is also well captured by the model.
 434 It is seen that measured results scatter around the simulated
 435 results, this could be explained by the fact that the fourth to
 436 sixth harmonics are of small magnitudes and may be influ-
 437 enced by noise during the measurements.

438 As shown in Fig. 4, waves become asymmetric in both
 439 vertical and horizontal directions when propagating over the
 440 submerged bar. The magnitudes of the vertical and hori-
 441 zontal asymmetry are measured by the skewness of the FSE
 442 and its Hilbert transform (Elgar and Guza, 1985), $\lambda_3(\eta)$ and
 443 $\lambda_3(\mathcal{H}(\eta))$ (known as asymmetry parameter, with \mathcal{H} denoting
 444 the Hilbert transform operator), respectively. The kurtosis is
 445 a measure of the extreme values in a time series: for irregu-
 446 lar waves, it is related to the occurrence probability of freak
 447 waves, and for regular waves, it is still an important index
 448 that characterizes wave nonlinearity. For a normalized ran-
 449 dom variable \bar{X} with a zero mean and a unit variance, the
 450 skewness, asymmetry and kurtosis parameters are defined
 451 respectively as:

$$\lambda_3(\bar{X}) = \langle \bar{X}^3 \rangle, \quad (44)$$

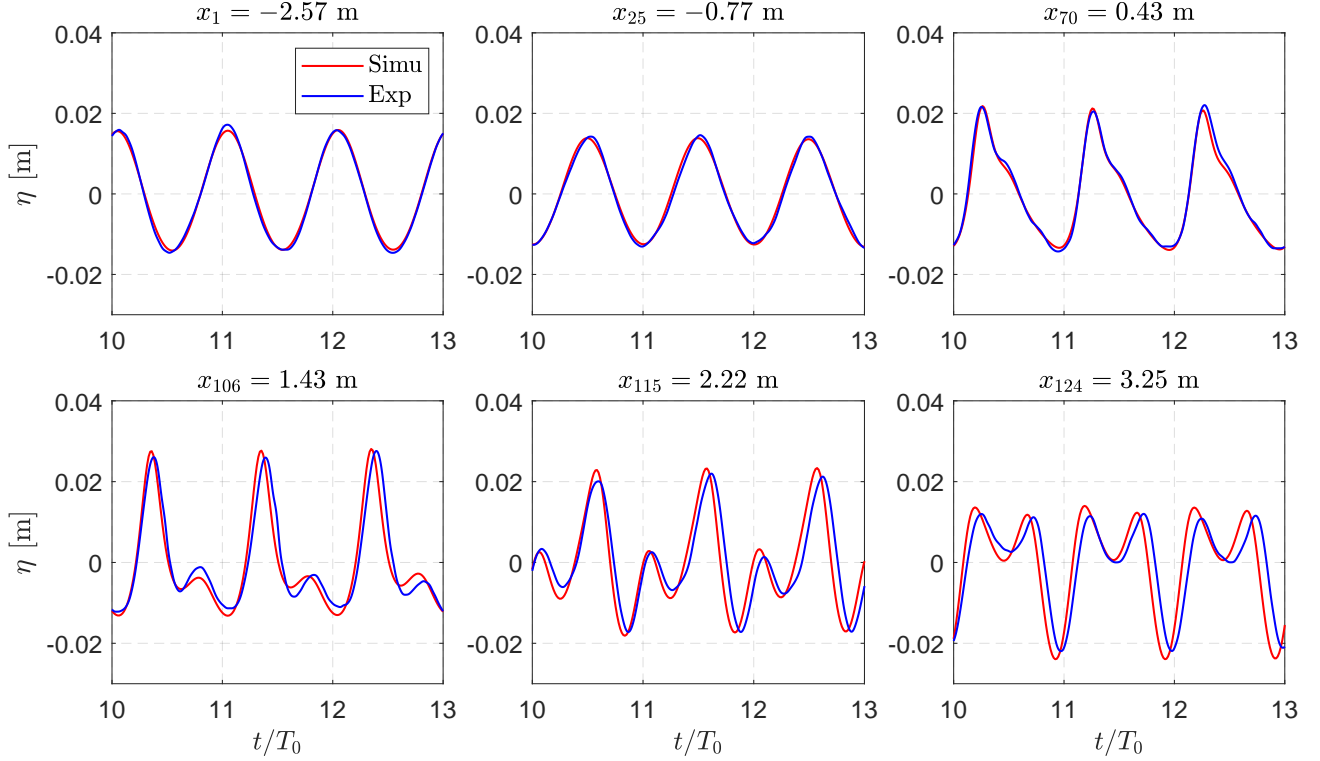


Figure 4: Comparison of measured and computed FSE temporal profiles at 6 locations for the nonlinear regular wave experiment of Lawrence et al. (2021a)

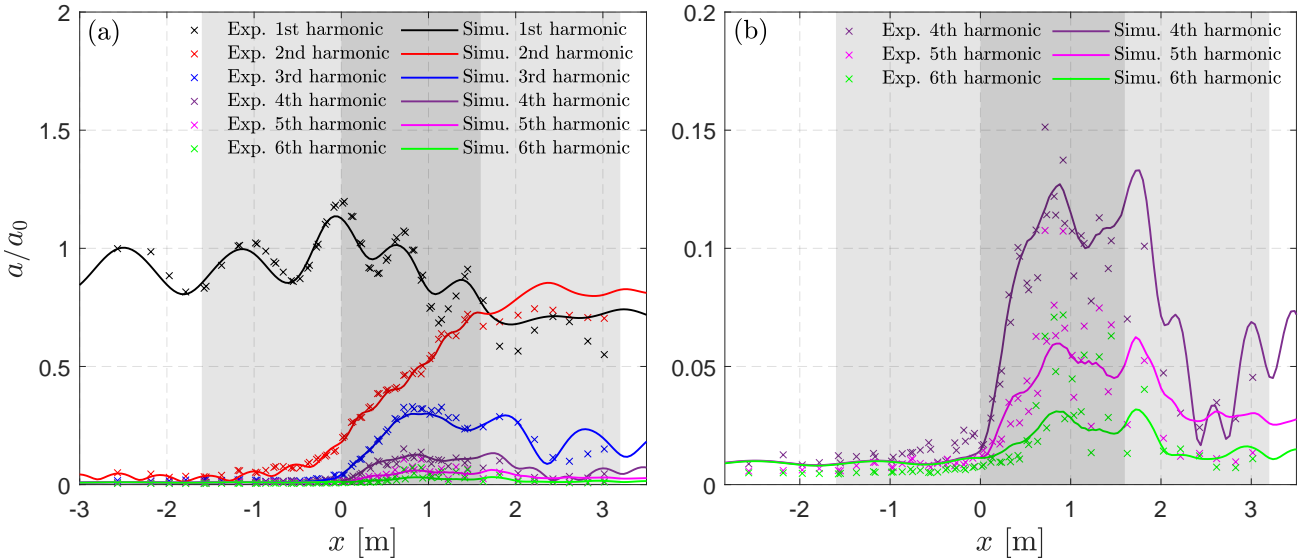


Figure 5: (a) Spatial evolution of measured and computed normalized amplitudes of the first 6 harmonics of the FSE for the nonlinear regular wave experiment of Lawrence et al. (2021a). (b) Close-up view of the fourth to sixth harmonics, with a reduced extent of the vertical axis. The gray areas indicate the extent of the submerged bar.

$$\lambda_3[H(\bar{X})] = \langle H(\bar{X})^3 \rangle, \quad (45)$$

$$\lambda_4(\bar{X}) = \langle \bar{X}^4 \rangle. \quad (46)$$

452 where $\langle \cdot \rangle$ denotes a mean operator, \bar{X} could be normalized
 453 FSE, velocity or acceleration. In this subsection, we take \bar{X}
 454 as $\bar{\eta}$. As we are discussing regular waves, in practice, the

455 skewness, asymmetry and kurtosis are computed for the averaged
 456 free surface profile over one wave period. In the linear framework,
 457 the skewness and asymmetry of a sinusoidal wave over one (or multiple)
 458 period(s) are 0, while the kurtosis is expected to be 1.5. The deviation
 459 of these parameters from their linear expectation is an indication of the magni-
 460

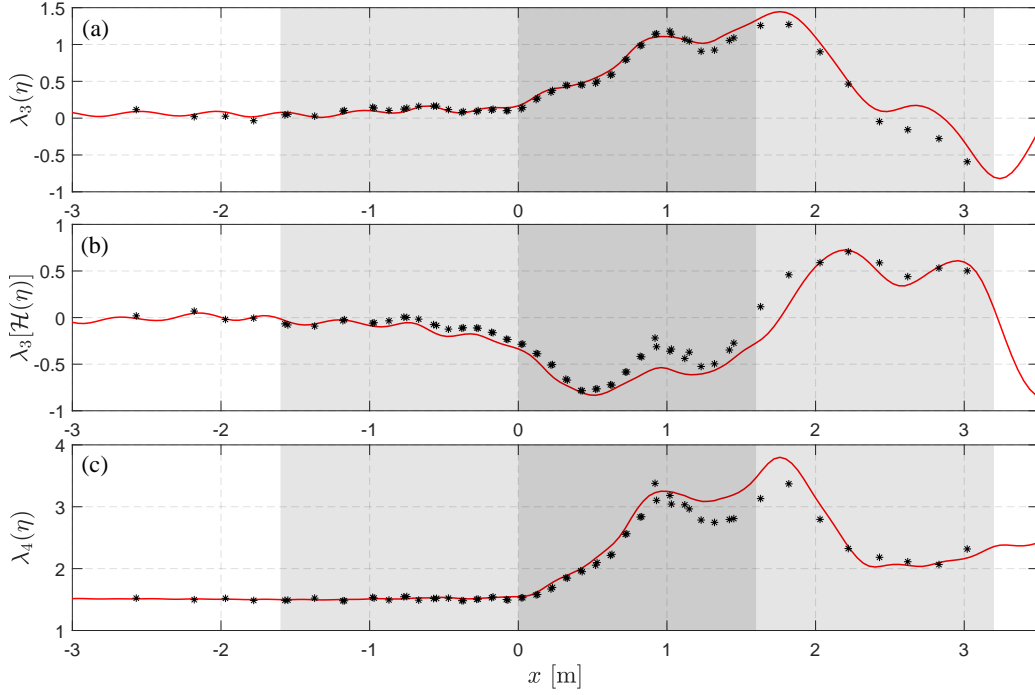


Figure 6: Spatial evolution of statistical moments of the measured and computed FSE for the nonlinear regular wave experiment of Lawrence et al. (2021a). The gray areas indicate the extent of the submerged bar.

461 tude of nonlinearity.

462 The evolution of skewness, asymmetry and kurtosis is
 463 shown in Fig. 6. It is observed that in Fig. 6(a), the skewness
 464 is increased over the bar crest, indicating waves with sharper
 465 crests and flatter troughs. It is then significantly decreased
 466 over the de-shoaling area, indicating that deep troughs and
 467 smaller crests develop in the wave profile in this area. In
 468 Fig. 6(b), the evolution of the asymmetry parameter indicates
 469 that the wave profile first leans forward then backward,
 470 and eventually restores a symmetric shape in the horizontal
 471 direction. Symmetrical wave profiles in both horizontal and
 472 vertical directions are not recovered after the de-shoaling
 473 zone, at least within the current spatial scale. In Fig. 6(c),
 474 the kurtosis of FSE shows a similar evolution trend as the skew-
 475 ness, it starts to deviate from the linear expectation 1.5 after
 476 the up-slope and increases significantly over the bar crest.
 477 Eventually, the kurtosis does not recover 1.5 after passing
 478 over the down-slope, and remains higher than this linear ex-
 479 pectation instead.

480 4.4. Orbital wave velocities at $z_0 = -0.05$ m

481 Fig. 7 shows the evolution profiles of $u(z_0)$ ($z_0 = -0.05$ m)
 482 at the same six positions as in Fig. 4, within again a time
 483 frame of $3T_0$. In general, the evolution of $u(z_0)$ profile is
 484 quite similar to that of η . The main difference between the
 485 evolution of $u(z_0)$ and η is that the contribution of the high-
 486 order harmonics is enhanced in the time series of $u(z_0)$. The
 487 enhancement is proportional to the wave number of high-
 488 order harmonics, and this explains the more pronounced sec-
 489 ondary peaks over and after the submerged bar (at locations
 490 of probes #29 and #30 for instance).

491 The harmonic analysis is performed for both the mea-
 492 sured and computed horizontal velocity $u(z_0)$. It is shown
 493 in Fig. 8(a) that the magnitudes and the evolution trends of
 494 $u(z_0)$ are reproduced by the model with high accuracy to the
 495 third order. In Fig. 8(b), the measured fourth to sixth-order
 496 harmonics of $u(z_0)$ are slightly lower than the model predic-
 497 tions. Again, the measured fourth to sixth harmonics show
 498 some oscillations around their mean levels, which could be
 499 related to digital noise. The spatial evolution of skewness,
 500 asymmetry and kurtosis is displayed in Fig. 9. The evolution
 501 trends of these statistical parameters of $u(z_0)$, in particular
 502 the skewness and asymmetry parameters, are very similar to
 503 those of η , indicating that the shapes of the horizontal ve-
 504 locity and FSE profiles evolve similarly.

505 To sum up, the various comparisons confirm the high
 506 accuracy of the W3D model in computing the horizontal ve-
 507 locity beneath nonlinear waves. In contrast, Lawrence et al.
 508 (2021a) indicate that they had to use two different models to
 509 simulate this case, namely a HOS-type model to predict the
 510 FSE field and a variational Boussinesq model to calculate
 511 the velocities in the fluid domain.

512 4.5. Eulerian wave accelerations at $z_0 = -0.05$ m

513 The horizontal acceleration $ax(z_0) = (\partial u / \partial t)|_{z=z_0}$ is not
 514 directly available from the measurements. Here, it is esti-
 515 mated by computing the time derivative of the measured
 516 $u(z_0)$ signal, using a centered FD scheme over a stencil of 5
 517 signal points (similar to eq. (36)). In the model, the accel-
 518 eration is directly evaluated using eq. (25). For comparison,
 519 we also computed it by deriving the simulated $u(z_0)$ time se-
 520 ries with respect to time, using the same FD scheme as for

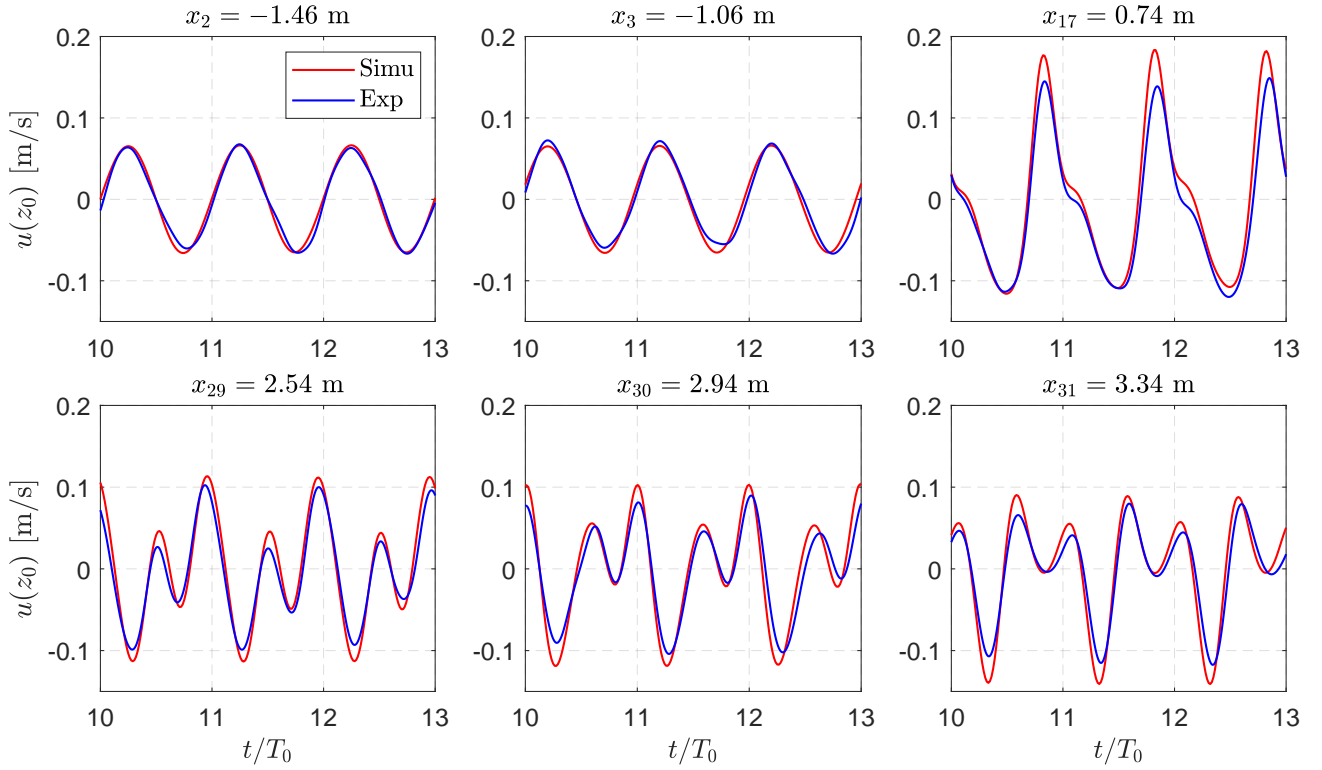


Figure 7: Comparison of measured and computed horizontal velocity $u(z_0)$ temporal profiles at 6 locations for the nonlinear regular wave experiment of Lawrence et al. (2021a).

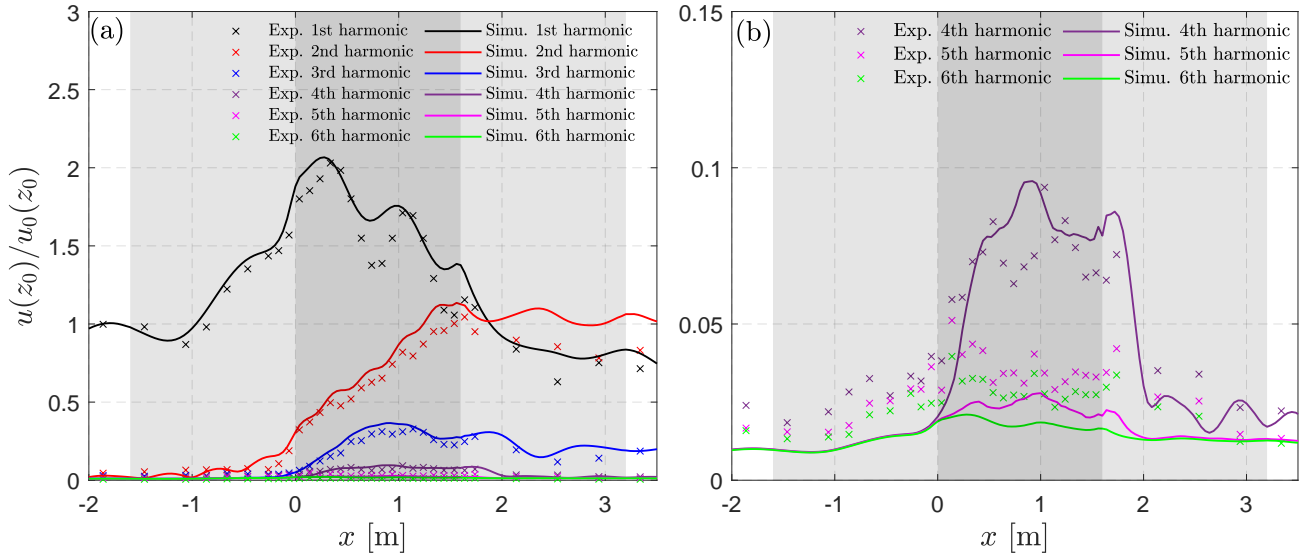


Figure 8: (a) Spatial evolution of normalized measured and computed amplitudes of the first 6 harmonics of the horizontal velocity $u(z_0)$ for the nonlinear regular wave experiment of Lawrence et al. (2021a). (b) Close-up view of the fourth to sixth harmonics, with a reduced extent of the vertical axis. The gray areas indicate the extent of the submerged bar.

521 the measurements.

522 In Fig. 10, the time profiles of $ax(z_0)$ at 6 locations (same
523 as in Fig. 7) are shown. In each panel of Fig. 10, the time
524 profiles of $ax(z_0)$ derived from the measured and simulated
525 $u(z_0)$ with FD method, and simulated $ax(z_0)$ evaluated with
526 eq. (25) are superimposed. It is seen that the $ax(z_0)$ directly

527 computed with eq. (25) is in good agreement with that calcu-
528 lated from $u(z_0)$ with the FD method, thus validating our im-
529 plementation of the acceleration computation. In the follow-
530 ing, unless otherwise stated, simulated $ax(z_0)$ refers to the
531 results obtained with eq. (25); measured $ax(z_0)$ refers to the
532 acceleration evaluated from the measured $u(z_0)$ with the FD

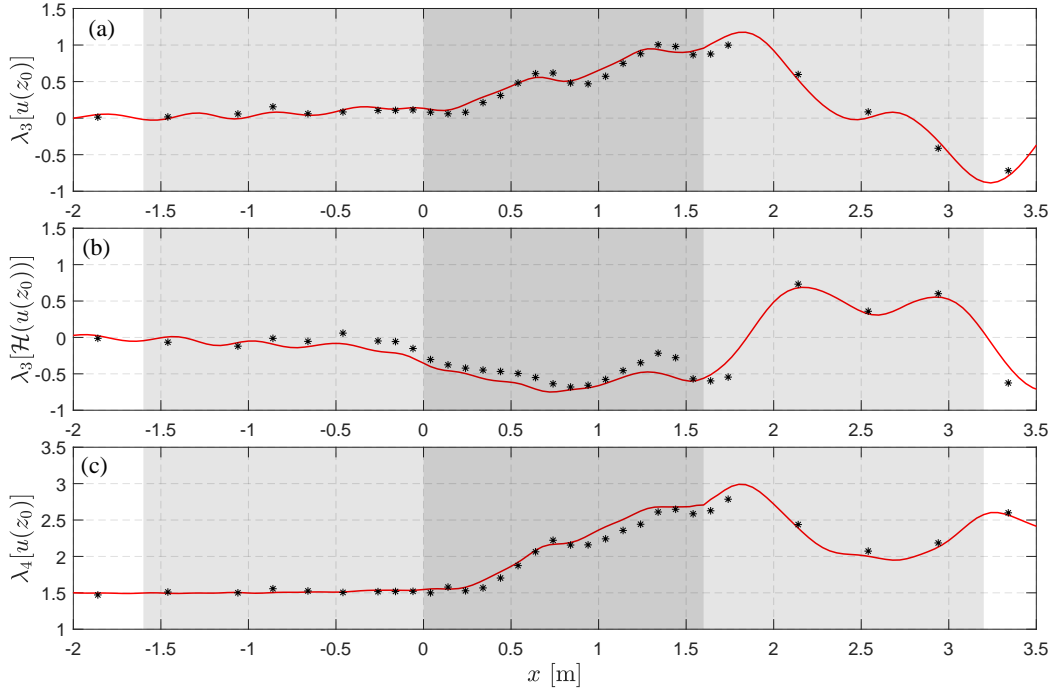


Figure 9: Spatial evolution of statistical moments of the measured and computed horizontal velocity $u(z_0)$ for the nonlinear regular wave experiment of Lawrence et al. (2021a). The gray areas indicate the extent of the submerged bar.

533 method. The agreement between the simulated and the measured $ax(z_0)$ is fairly good throughout the domain, although
 534 some differences in the magnitude of crests and troughs develop after the up-slope.
 535

536
 537 The harmonic analysis is performed for both the simulated and measured $ax(z_0)$ time series, with the evolution of
 538 the amplitudes of the first six harmonics displayed in Fig. 11. In Fig. 11(a), it is seen that the agreement between the simulated and measured results is reasonable. Yet, the second-
 539 order harmonic is overestimated in the simulation starting around $x = 0.5$ m, leading to higher crests and deeper troughs in the simulated $ax(z_0)$ time series. It is noted that the amplitudes of the super-harmonics (second and higher orders)
 540 are increasing when comparing the spectral evolution of η , $u(z_0)$, and $ax(z_0)$ in space. This is the result of the free super-harmonics excited by the shoal, which produce larger
 541 amplitudes of velocity and acceleration in comparison to the bound components.
 542
 543
 544
 545
 546
 547
 548
 549
 550

551 In Fig. 12, the spatial evolution of skewness, asymmetry, and kurtosis of $ax(z_0)$ are shown. It is seen that the simulated results are in very good agreement with the measured
 552 results. The evolution trends of these parameters are evidently different from those for η and $u(z_0)$, especially after the up-slope: the local peak of the skewness appears much
 553 closer to the end of the up-slope, and the asymmetry parameter is positive over the bar. It is speculated that the differences are related to the phase differences between horizontal
 554 acceleration and horizontal velocity. The kurtosis increases rapidly after the shoal and remains at a high level over the bar and a short distance over the de-shoaling zone. The evolu-
 555 tion trends of these parameters are in line with the indication
 556
 557
 558
 559
 560
 561
 562
 563

of stronger second- and higher-order harmonics observed in
 Fig. 11. It is stressed that the kurtosis after de-shoal is hard
 to predict, and the W3D model performs well for that pur-
 pose, providing excellent prediction of not only the kurtosis
 of FSE but also of the kinematics underneath.

5. Conclusion

In this study, new formulations of the particle kinematics, namely orbital velocities and accelerations, have been developed in the FNPF wave model Whispers3D, which uses a basis of orthogonal Chebyshev polynomials of the first kind to project the vertical structure of the velocity potential. With the potential expressed (and approximated) in a polynomial form as given by eq. (8) where the main unknowns are then the a_n coefficients ($n = 0, 1, \dots, N_T$), the expressions of particle kinematics could be derived explicitly. The estimation of horizontal and vertical velocities involves the spatial derivation of the potential in the corresponding direction, which can be obtained either analytically or with the FD method without additional information in the time domain. However, the estimation of particle accelerations involves the time derivative of the velocity components. In the model, this requires the computation of the time derivatives of a_n coefficients, which can be obtained by using either a backward (in time) FD scheme in the course of the simulation or a centered FD scheme after the completion of the run. The results shown here are obtained with a four-point backward FD scheme (i.e. using the value at the current time plus the ones at the three previous time steps). The accuracy and efficiency in the computation of the particle kinemat-

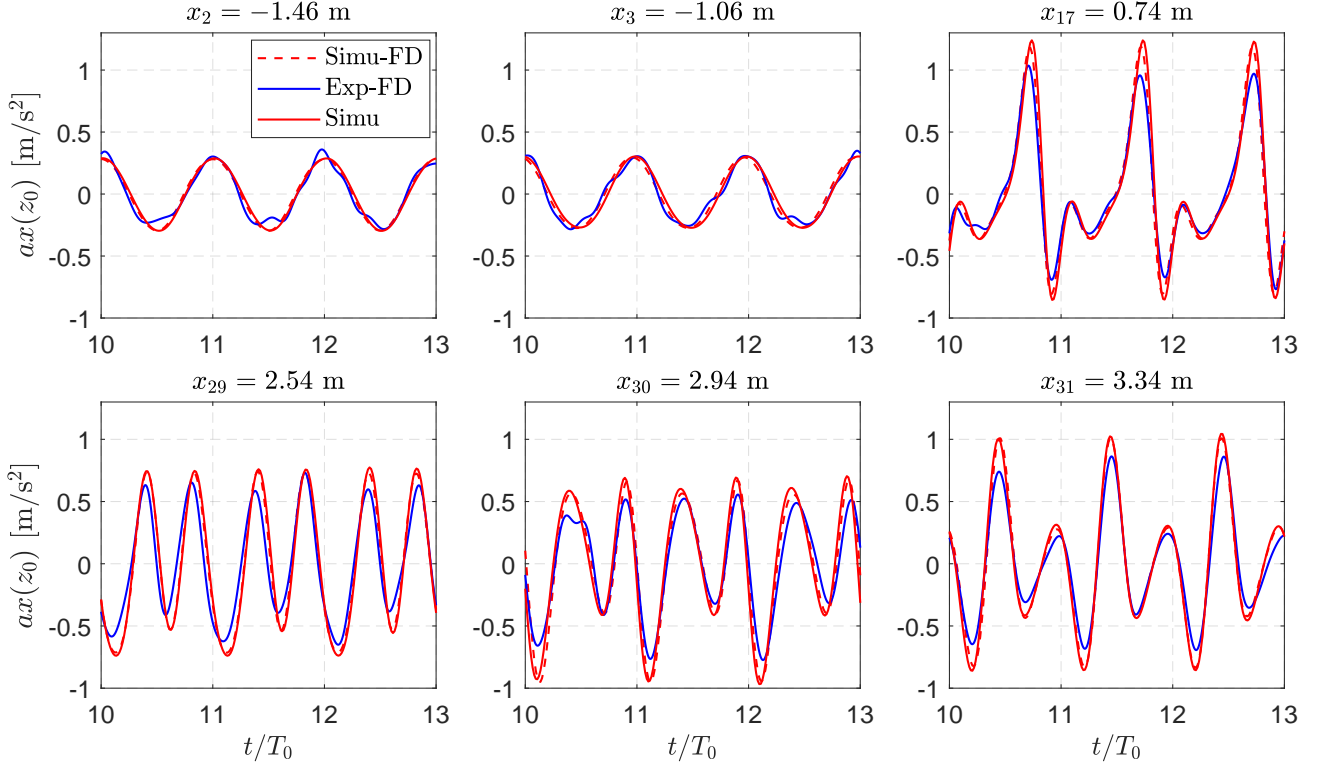


Figure 10: Comparison of the temporal profiles of the horizontal acceleration $ax(z_0)$, between the time-derivative of the measured horizontal velocity and the computed acceleration given by the model at 6 locations for the nonlinear regular wave experiment of Lawrence et al. (2021a).

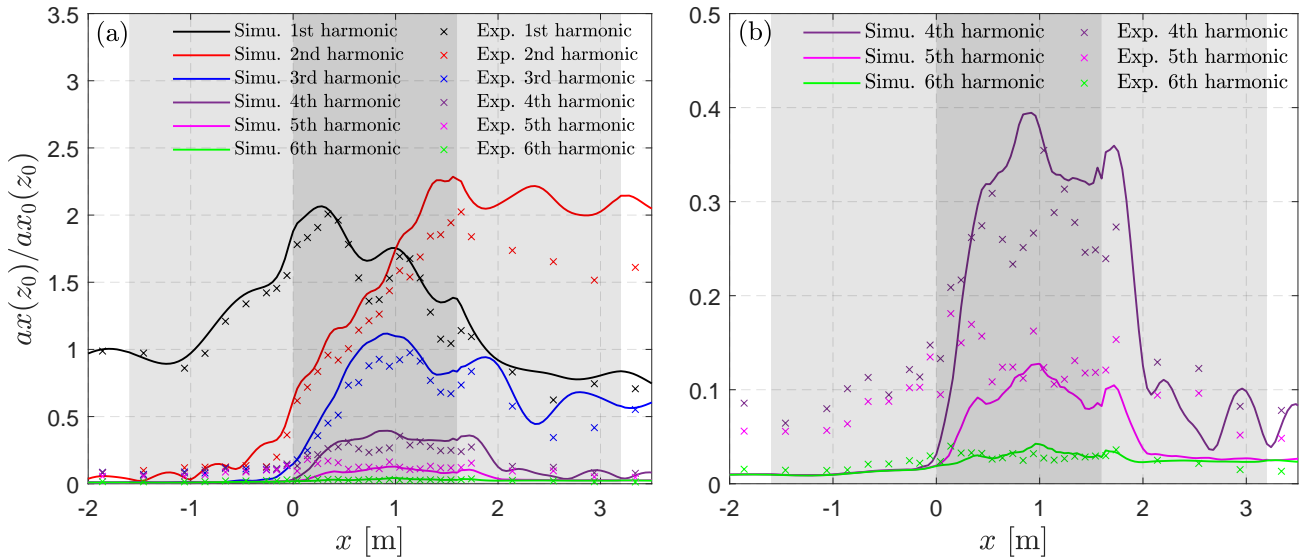


Figure 11: (a) Spatial evolution of normalized measured and computed amplitudes of the first 6 harmonics of the horizontal acceleration $ax(z_0) = u_x(z_0)$ for the nonlinear regular wave experiment of Lawrence et al. (2021a). (b) Close-up view of the fourth to sixth harmonics, with a reduced extent of the vertical axis. The gray areas indicate the extent of the submerged bar.

593 ics are governed by the maximum degree of the Chebyshev
594 polynomial N_T in eq. (8).

595 We have then demonstrated the capability and accuracy
596 of the model to simulate kinematics beneath regular non-
597 linear wave trains in either uniform or variable water depth

through comparisons with an analytical solution and exper- 598
imental measurements. In the case of regular waves propa- 599
gating over a flat bottom, a deep-water ($kh = 2\pi$) and nearly- 600
breaking ($ka = 0.40$) wave condition is tested. With $(\eta, \tilde{\phi})$ 601
calculated from the SF theory and imposed as initial condi- 602

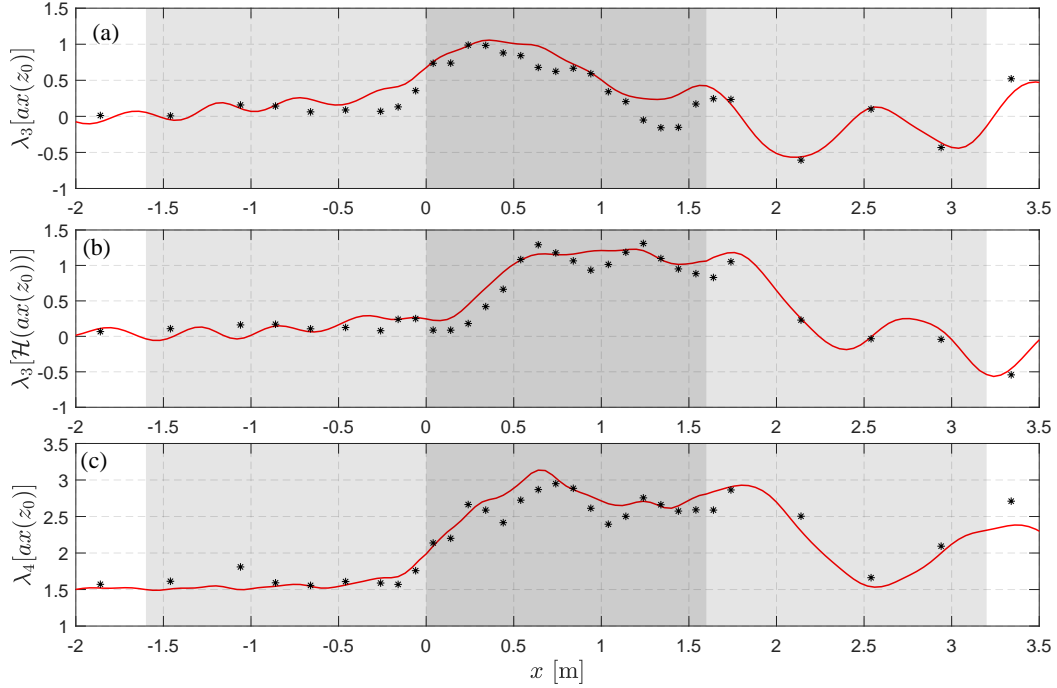


Figure 12: Spatial evolution of statistical moments of the measured and computed horizontal acceleration $ax(z_0) = u_t(z_0)$ for the nonlinear regular wave experiment of Lawrence et al. (2021a). The gray areas indicate the extent of the submerged bar.

603 tions on the free surface, the W3D model successfully computes
 604 the kinematics in the whole fluid domain beneath the
 605 free surface. The errors of computed FSE, velocities, and
 606 accelerations in comparison to the reference SF solution occur
 607 mainly beneath the wave crest, yet remain very low (below
 608 0.1% for velocities and 1% for the accelerations) with
 609 $N_T = 11$. An even better agreement could be achieved with
 610 a further increase of N_T , at the cost of an additional compu-
 611 tational burden though. In the case of regular waves propa-
 612 gating over an uneven bottom (submerged trapezoidal bar),
 613 the experimental test reported in Lawrence et al. (2021b)
 614 is reproduced with the W3D model. A Fourier analysis of
 615 both measured and simulated time series shows a very good
 616 to excellent agreement between simulated results and mea-
 617 surements achieved for the amplitudes up to the sixth-order
 618 super-harmonics of FSE, velocities, and accelerations. The
 619 spatial evolution trends of statistical parameters describing
 620 wave nonlinearity of the kinematics (skewness, asymmetry,
 621 kurtosis) are also very well described by the model.

622 As a general conclusion, the W3D model is capable of
 623 computing the kinematics beneath strongly nonlinear waves
 624 very accurately under the framework of potential wave theo-
 625 ry. In comparison to other existing models, some advan-
 626 tages of the W3D model can be summarized as follows:

- 627 1. It is a single-layer model that can handle nonlinear
 628 waves in a broad range of relative water depth, whereas
 629 a multi-layer approach is often required for higher-
 630 order Boussinesq-type models aiming at a similar range
 631 of application (see e.g. Liu and Fang, 2016; Fang et al.,
 632 2022);
- 633 2. With the newly developed compact formulas, namely

634 eqs. (17)–(18) for the velocity components and eqs. (25)–
 635 (26) for the acceleration components, explicit high-
 636 order polynomial expressions are available to compute
 637 the wave-induced kinematics at any point at or below
 638 the free surface.

- 639 3. The model is free from any singularity issue when
 640 computing the kinematics throughout the water col-
 641 umn, which is different from other approaches as the
 642 BEM for instance, in which singularities appear for
 643 nodes located at the free surface (see e.g. Lefe et al.,
 644 1980; Wang and Tsay, 2005).
- 645 4. It can describe kinematics beneath strongly nonlin-
 646 ear waves with high accuracy. In contrast, tackling
 647 the same question with the HOS modelling framework
 648 can be of larger error close to the free surface, as ob-
 649 served by Lawrence et al. (2021a) for instance, who
 650 had to rely on a two-model approach, namely a HOS
 651 model for simulating the wave field evolution and
 652 a variational Boussinesq model to subsequently com-
 653 pute wave kinematics.

654 In the companion Part II article (Zhang et al., 2024), we pro-
 655 vide additional validation of the numerical model against ir-
 656 regular wave experiments performed by Trulsen et al. (2020)
 657 in which the same bottom profile as in section 4 was adopted.
 658 The combination of measured and numerically simulated long
 659 time-series of wave kinematics permits studying the statisti-
 660 cal distributions of particle velocities and accelerations, and
 661 proposing and validating a new statistical model of log-normal
 662 type for these kinematics variables.

CRedit authorship contribution statement

Michel Benoit: Writing – Review & Editing, Writing – Original Draft, Supervision, Methodology, Conceptualization. **Jie Zhang:** Writing – Original Draft, Investigation, Funding acquisition, Formal analysis. **Yuxiang Ma:** Resources, Writing – Review & Editing, Supervision.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Acknowledgments

This work was supported by the National Natural Science Foundation of China (Grant No. 52101301), and the China Postdoctoral Science Foundation (Grants No. 2023T160078, 2021M690523). The authors would like to express their gratitude to Christopher Lawrence, Karsten Trulsen, and Odin Gramstad for kindly sharing their experimental dataset in Lawrence et al. (2021a).

References

- Aggarwal, A., Chella, M.A., Kamath, A., Bihs, H., Arntsen, Ø.A., 2016. Irregular wave forces on a large vertical circular cylinder. *Energy Procedia* 94, 504–516. doi:10.1016/j.egypro.2016.09.223.
- Antuono, M., Colagrossi, A., Marrone, S., Lugni, C., 2011. Propagation of gravity waves through an SPH scheme with numerical diffusive terms. *Comput. Phys. Commun.* 182, 866–877. doi:10.1016/j.cpc.2010.12.012.
- Bassi, C., Bonaventura, L., Busto, S., Dumbser, M., 2020. A hyperbolic reformulation of the Serre-Green-Naghdi model for general bottom topographies. *Comput. Fluids* 212, 104716. doi:10.1016/j.compfluid.2020.104716.
- Belibassakis, K.A., Athanassoulis, G.A., 2011. A coupled-mode system with application to nonlinear water waves propagating in finite water depth and in variable bathymetry regions. *Coast. Eng.* 58, 337–350. doi:10.1016/j.coastaleng.2010.11.007.
- Benoit, M., Raoult, C., Yates, M.L., 2017. Analysis of the linear version of a highly dispersive potential water wave model using a spectral approach in the vertical. *Wave Motion* 74, 159–181. doi:10.1016/j.wavemoti.2017.07.002.
- Berkhoff, J., 1972. Mathematical models for simple harmonic linear water waves: wave diffraction and refraction. *Coast. Eng. Proc. (ICCE'1972)* 1(13), 23. doi:10.9753/icce.v13.23.
- Bingham, H.B., Madsen, P.A., Fuhrman, D.R., 2009. Velocity potential formulations of highly accurate Boussinesq-type models. *Coast. Eng.* 56, 467–478. doi:10.1016/j.coastaleng.2008.10.012.
- Boussinesq, J., 1872. Théorie des ondes et des remous qui se propagent le long d'un canal rectangulaire horizontal, en communiquant au liquide contenu dans ce canal des vitesses sensiblement pareilles de la surface au fond. *J. Math. Pures Appl.* 17, 55–108.
- Craig, W., Sulem, C., 1993. Numerical simulation of gravity waves. *J. Comput. Phys.* 108, 73–83. doi:10.1006/jcph.1993.1164.
- Dalrymple, R.A., Rogers, B.D., 2006. Numerical modeling of water waves with the SPH method. *Coast. Eng.* 53, 141–147. doi:10.1016/j.coastaleng.2005.10.004.
- Decorte, G., Toffoli, A., Lombaert, G., Monbaliu, J., 2021. On the use of a domain decomposition strategy in obtaining response statistics in non-Gaussian seas. *Fluids* 6, 28. doi:10.3390/fluids6010028.
- Deng, Y., Zhu, C., Wang, Z., 2023. A comparative study of wave kinematics and inline forces on vertical cylinders under Draupner-type freak waves. *Ocean Eng.* 288, 115959. doi:10.1016/j.oceaneng.2023.115959.

- Didenkulova, E., Didenkulova, I., Medvedev, I., 2023. Freak wave events in 2005–2021: statistics and analysis of favourable wave and wind conditions. *Nat. Hazards Earth Syst. Sci.* 23, 1653–1663. doi:10.5194/nhess-23-1653-2023.
- Didenkulova, E.G., Pelinovsky, E.N., 2020. Freak waves in 2011–2018. *Dokl. Earth Sci.* 491, 187–190. doi:10.1134/s1028334x20030046.
- Didenkulova, I., Pelinovsky, E., 2016. On shallow water rogue wave formation in strongly inhomogeneous channels. *J. Phys. A: Math. Theor.* 49, 194001. doi:10.1088/1751-8113/49/19/194001.
- Dommermuth, D., 2000. The initialization of nonlinear waves using an adjustment scheme. *Wave Motion* 32, 307–317. doi:10.1016/s0165-2125(00)00047-0.
- Dysthe, K.B., 1979. Note on a modification to the nonlinear Schrödinger equation for application to deep water waves. *Proc. R. Soc. A: Math. Phys. Eng.* 369, 105–114. doi:10.1098/rspa.1979.0154.
- Elfrink, B., Baldock, T., 2002. Hydrodynamics and sediment transport in the swash zone: a review and perspectives. *Coast. Eng.* 45, 149–167. doi:10.1016/s0378-3839(02)00032-7.
- Elgar, S., Guza, R.T., 1985. Observations of bispectra of shoaling surface gravity waves. *J. Fluid Mech.* 161, 425–448. doi:10.1017/S0022112085003007.
- Fang, K., Liu, Z., Wang, P., Wu, H., Sun, J., Yin, J., 2022. Modeling solitary wave propagation and transformation over complex bathymetries using a two-layer Boussinesq model. *Ocean Eng.* 265, 112549. doi:10.1016/j.oceaneng.2022.112549.
- Fenton, J.D., 1990. *The Sea - Vol. 9: Ocean Engineering Science, Part A*. Wiley. chapter 1. Nonlinear wave theories. pp. 3–25.
- Fochesato, C., Dias, F., 2006. A fast method for nonlinear three-dimensional free-surface waves. *Proc. R. Soc. A: Math. Phys. Eng.* 462, 2715–2735. doi:10.1098/rspa.2006.1706.
- Fredsøe, J., Deigaard, R., 1992. *Mechanics of Coastal Sediment Transport*. Advanced Series on Ocean Engineering: Vol. 3. World Scientific. doi:10.1142/1546.
- Freilich, M.H., Guza, R.T., 1984. Nonlinear effects on shoaling surface gravity waves. *Philos. Trans. A: Math. Phys. Eng. Sci.* 311, 1–41. doi:10.1098/rsta.1984.0019.
- Gouin, M., Ducrozet, G., Ferrant, P., 2016. Development and validation of a non-linear spectral model for water waves over variable depth. *Eur. J. Mech. B Fluids* 57, 115–128. doi:10.1016/j.euromechflu.2015.12.004.
- Green, A.E., Laws, N., Naghdi, P.M., 1974. On the theory of water waves. *Proc. R. Soc. A: Math. Phys. Eng.* 338, 43–55. doi:10.1098/rspa.1974.0072.
- Grilli, S.T., Skourup, J., Svendsen, I.A., 1989. An efficient boundary element method for nonlinear water waves. *Eng. Anal. Boundary Elem.* 6, 97–107. doi:10.1016/0955-7997(89)90005-2.
- Harris, J.C., Dombre, E., Benoit, M., Grilli, S.T., 2014. Fast integral equation methods for fully nonlinear water wave modeling, in: *Proc. 24th Int. Ocean Polar Eng. Conf. (ISOPE'2014)*, Busan, Korea, 15–20 June 2014, pp. ISOPE–I–14–449.
- Hasimoto, H., Ono, H., 1972. Nonlinear modulation of gravity waves. *J. Phys. Soc. Japan* 33, 805–811. doi:10.1143/JPSJ.33.805.
- Higuera, P., Lara, J.L., Losada, I.J., 2013. Simulating coastal engineering processes with OpenFOAM®. *Coast. Eng.* 71, 119–134. doi:10.1016/j.coastaleng.2012.06.002.
- Jacobsen, N.G., van Gent, M.R., Wolters, G., 2015. Numerical analysis of the interaction of irregular waves with two dimensional permeable coastal structures. *Coast. Eng.* 102, 13–29. doi:10.1016/j.coastaleng.2015.05.004.
- Kriebel, D.L., 1998. Nonlinear wave interaction with a vertical circular cylinder: wave forces. *Ocean Eng.* 7, 597–605.
- Lafe, O.E., Liu, P.L.F., Liggett, J.A., Cheng, A.H.D., Montes, J.S., 1980. Singularities in Darcy flow through porous media. *J. Hydraul. Div.* 106, 977–997. doi:10.1061/jyceaj.0005457.
- Larsen, B.E., Fuhrman, D.T., Roenby, J., 2019. Performance of interFoam on the simulation of progressive waves. *Coast. Eng. J.* 61, 380–400. doi:10.1080/21664250.2019.1609713.
- Lawrence, C., Gramstad, O., Trulsen, K., 2021a. Variational Boussinesq model for kinematics calculation of surface gravity waves over

- 789 bathymetry. *Wave Motion* 100, 102665. doi:10.1016/j.wavemoti.2020.
790 102665.
- 791 Lawrence, C., Trulsen, K., Gramstad, O., 2021b. Statistical properties of
792 wave kinematics in long-crested irregular waves propagating over non-
793 uniform bathymetry. *Phys. Fluids* 33, 046601. doi:10.1063/5.0047643.
- 794 Li, Z., Tang, T., Li, Y., Draycott, S., van den Bremer, T.S., Adcock, T.A.A.,
795 2023. Wave loads on ocean infrastructure increase as a result of waves
796 passing over abrupt depth transitions. *J. Ocean Eng. Mar. Energy* 9,
797 309–317. doi:10.1007/s40722-022-00269-4.
- 798 Liu, Z., Fang, K., 2016. A new two-layer Boussinesq model for coastal
799 waves from deep to shallow water: Derivation and analysis. *Wave Mo-
800 tion* 67, 1–14. doi:10.1016/j.wavemoti.2016.07.002.
- 801 Madsen, P.A., Fuhrman, D.R., Wang, B., 2006. A Boussinesq-type method
802 for fully nonlinear waves interacting with a rapidly varying bathymetry.
803 *Coast. Eng.* 53, 487–504. doi:10.1016/j.coastaleng.2005.11.002.
- 804 Madsen, P.A., Schäffer, H.A., 1998. Higher-order Boussinesq-type equa-
805 tions for surface gravity waves: derivation and analysis. *Philos. Trans.
806 Royal Soc. A* 356, 3123–3181. doi:10.1098/rsta.1998.0309.
- 807 Papoutsellis, C.E., Yates, M.L., Simon, B., Benoit, M., 2019. Modelling of
808 depth-induced wave breaking in a fully nonlinear free-surface potential
809 flow model. *Coast. Eng.* 154, 103579. doi:10.1016/j.coastaleng.2019.
810 103579.
- 811 Paulsen, B.T., Bredmose, H., Bingham, H.B., 2014. An efficient domain
812 decomposition strategy for wave loads on surface piercing circular cylin-
813 ders. *Coast. Eng.* 86, 57–76. doi:10.1016/j.coastaleng.2014.01.006.
- 814 Porter, D., 2003. The mild-slope equations. *J. Fluid Mech.* 494, 51–63.
815 doi:10.1017/s0022112003005846.
- 816 Raoult, C., Benoit, M., Yates, M.L., 2016. Validation of a fully nonlinear
817 and dispersive wave model with laboratory non-breaking experiments.
818 *Coast. Eng.* 114, 194–207. doi:10.1016/j.coastaleng.2016.04.003.
- 819 Raoult, C., Benoit, M., Yates, M.L., 2019. Development and validation of
820 a 3D RBF-spectral model for coastal wave simulation. *J. Comp. Phys.*
821 378, 278–302. doi:10.1016/j.jcp.2018.11.002.
- 822 Shi, J., Feng, X., Toumi, R., Zhang, C., Hodges, K., Tao, A., Zhang, W.,
823 Zheng, J., 2024. Global increase in tropical cyclone ocean surface waves.
824 *Nat. Commun.* 15, 174. doi:10.1038/s41467-023-43532-4.
- 825 Simon, B., Papoutsellis, C.E., Benoit, M., Yates, M.L., 2019. Comparing
826 methods of modeling depth-induced breaking of irregular waves with a
827 fully nonlinear potential flow approach. *J. Ocean Eng. Mar. Energy* 5,
828 365–383. doi:10.1007/s40722-019-00154-7.
- 829 Stansberg, C.T., Huse, E., Krokstad, J.R., Lehn, E., 1995. Experimental
830 study of non-linear loads on vertical cylinders in steep random waves,
831 in: *Proc. 5th Int. Ocean Polar Eng. Conf. (ISOPE'1995)*, The Hague,
832 The Netherlands, 11–16 June 1995, pp. ISOPE-I-95–013.
- 833 Tian, Y., Sato, S., 2008. A numerical model on the interaction between
834 nearshore nonlinear waves and strong currents. *Coast. Eng. J.* 50, 369–
835 395. doi:10.1142/s0578563408001879.
- 836 Trulsen, K., Raustøl, A., Jorde, S., Rye, L.B., 2020. Extreme wave statistics
837 of long-crested irregular waves over a shoal. *J. Fluid Mech.* 882, R2.
838 doi:10.1017/jfm.2019.861.
- 839 Vested, M.H., Carstensen, S., Christensen, E.D., 2020. Experimental study
840 of wave kinematics and wave load distribution on a vertical circular
841 cylinder. *Coast. Eng.* 157, 103660. doi:10.1016/j.coastaleng.2020.
842 103660.
- 843 Wang, J., Tsay, T.K., 2005. Analytical evaluation and application of the
844 singularities in boundary element method. *Eng. Anal. Bound. Elem.* 29,
845 241–256. doi:10.1016/j.enganabound.2004.12.008.
- 846 Wang, Z., Zou, Q., Reeve, D., 2009. Simulation of spilling breaking waves
847 using a two phase flow CFD model. *Comput. Fluids* 38, 1995–2005.
848 doi:10.1016/j.compfluid.2009.06.006.
- 849 Wilson, J.F., 2002. *Dynamics of Offshore Structures*. Wiley.
- 850 Yates, M.L., Benoit, M., 2015. Accuracy and efficiency of two numerical
851 methods of solving the potential flow problem for highly nonlinear and
852 dispersive water waves. *Int. J. Numer. Meth. Fluids* 77, 616–640. doi:10.
853 1002/flid.3992.
- 854 Zakharov, V.E., 1968. Stability of periodic waves of finite amplitude on
855 the surface of a deep fluid. *J. Appl. Mech. Tech. Phys.* 9, 190–194.
856 doi:10.1007/bf00913182.
- Zelt, J.A., Gudmestad, O.T., Skjelbreia, J.E., 1995. Fluid accelerations
under irregular waves. *Appl. Ocean Res.* 17, 43–54. doi:10.1016/
0141-1187(94)00019-j.
- Zhang, J., Benoit, M., 2021. Wave–bottom interaction and extreme wave
statistics due to shoaling and de-shoaling of irregular long-crested wave
trains over steep seabed changes. *J. Fluid Mech.* 912, A28. doi:10.1017/
jfm.2020.1125.
- Zhang, J., Benoit, M., Ma, Y., 2022. Equilibration process of out-of-
equilibrium sea-states induced by strong depth variation: Evolution of
coastal wave spectrum and representative parameters. *Coast. Eng.* 174,
104099. doi:10.1016/j.coastaleng.2022.104099.
- Zhang, J., Ma, Y., Benoit, M., 2024. Kinematics of nonlinear waves over
variable bathymetry. Part II: Statistical distributions of orbital veloci-
ties and accelerations in irregular long-crested seas. *Coast. Eng.* 193,
104589.